Using randomly assigned normally distributed draws for estimating Maximum Simulated Likelihood

Otto von Guericke University Magdeburg

23 August 2019

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NEW ZEALAND WORK RESEARCH INSTITUTE

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Work in progress

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Motivation:

- Maximum Simulated Likelihood (MSL): integrating out (multivariate) normal densities
- Approach: simulating likelihood and averaging over these
- What we need: draws from standard uniform density, interval [0,1)
- Draws are taken from:
  - ➢ Pseudorandom number generator (Stata: *runiform()*)
  - ➢ Quasi-random number generator using prime numbers (Halton draws)
- Today: randomly assigned normally distributed draws (RAND)

Maximum Simulated Likelihood:

- Popularity increased with computational power
- Advantage: flexibility in modelling multivariate normal densities:
  '*it can be readily applied in conjunction with almost any joint distribution of random parameters*' (Hole & Yoo 2017, p. 998)
- Attention in economic literature:
  - Mixed logit models / random parameter logit
  - Cappellari & Jenkins (2008): Low pay – panel retention – employment
  - Stewart (2007): heterogeneous slope model
  - Cai et al. (2018): NILF, unemployment, self-employment, low pay and higher pay

Maximum Simulated Likelihood:

- Problem: '*likelihood is a multi-dimensional integral which has no closed form expression and needs to be numerically approximated*' (Hole & Yoo 2017, p. 997)
- Why is integrating out possible?
  - Sample: $i = 1, \dots N$ individuals, observed $t = 1, \dots, T$ periods
  - Example:

    $$y_{it} = \mathbf{1}(x_{it}'\beta + \alpha_i + u_{it} > 0)$$

    with $\alpha_i \sim \text{iid } N(0, \sigma_\alpha^2)$ and independent of x_{it} and u_{it} for all i, t.
  - $u_{it} \sim N(0, \sigma_u^2) \rightarrow$ normalization required, convenient one is $\sigma_u^2 = 1$
  - $P_{it}(\alpha^*) = \Phi(x_{it}'\beta + \sigma_\alpha \alpha^*)$ and $\alpha^* = \alpha/\sigma_\alpha$

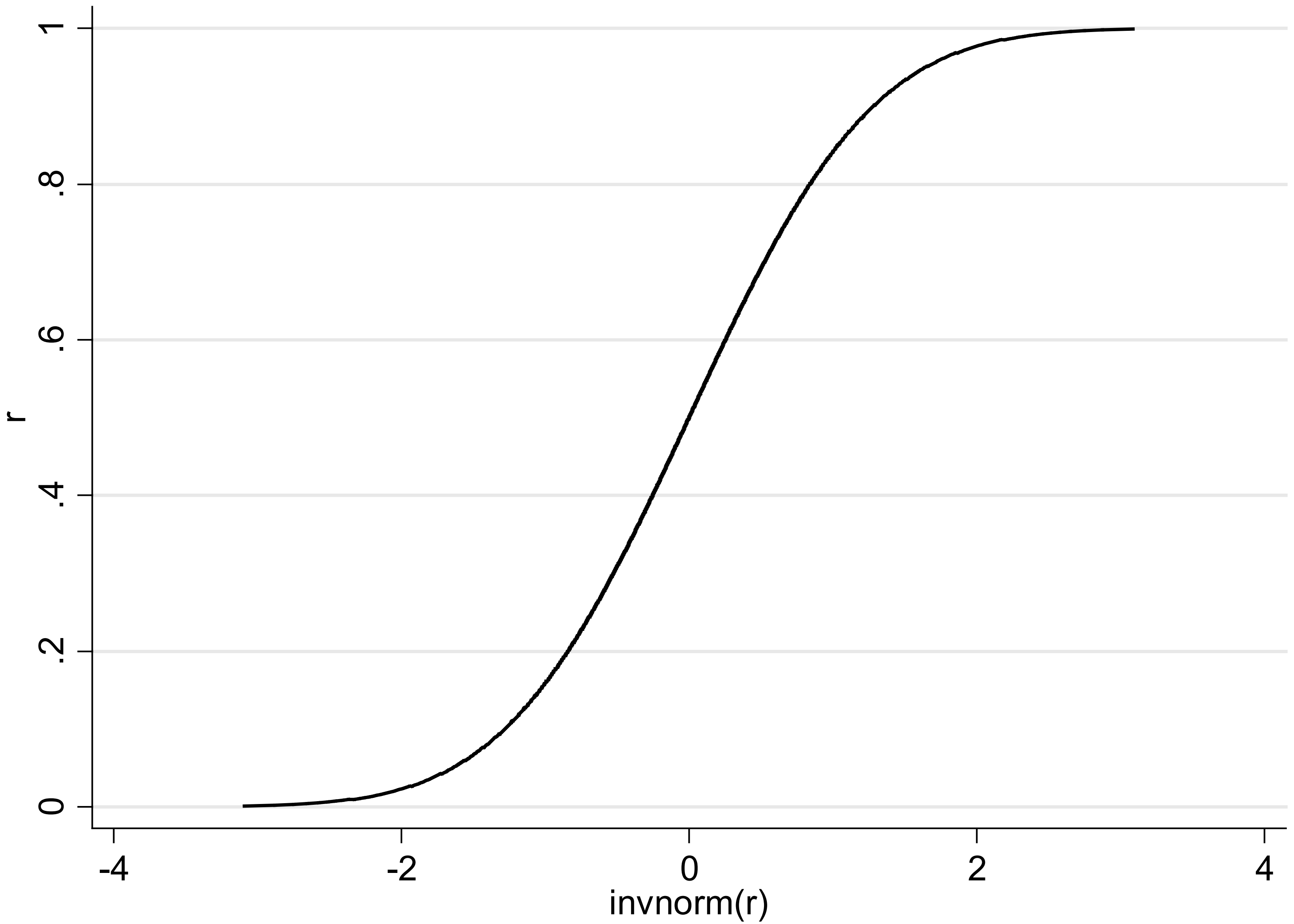
Maximum Simulated Likelihood:

- Concept of MSL – in plain English
  - α_i captures individual-specific time-invariant differences like motivation/ability (by definition: completely exogenous)
  - Motivation/ability ranges from very low to very high
  - For each individual, every possible scenario from very low level of motivation/ability to very high level of motivation/ability is calculated
  - There is no link between the individuals' level of motivation/ability; thus, during each scenario, α_i is normally distributed

Maximum Simulated Likelihood:

- Concept of MSL:
  - Sample size: $N \times T$
  - K-parameters that need to be integrated out $(k = 1, \ldots, K)$
  - For each parameter, take $d = 1, \ldots, D$ draws from a standard uniform density
  - Transform by the inverse standard normal distribution (Φ^{-1})

2. Maximum Simulated Likelihood



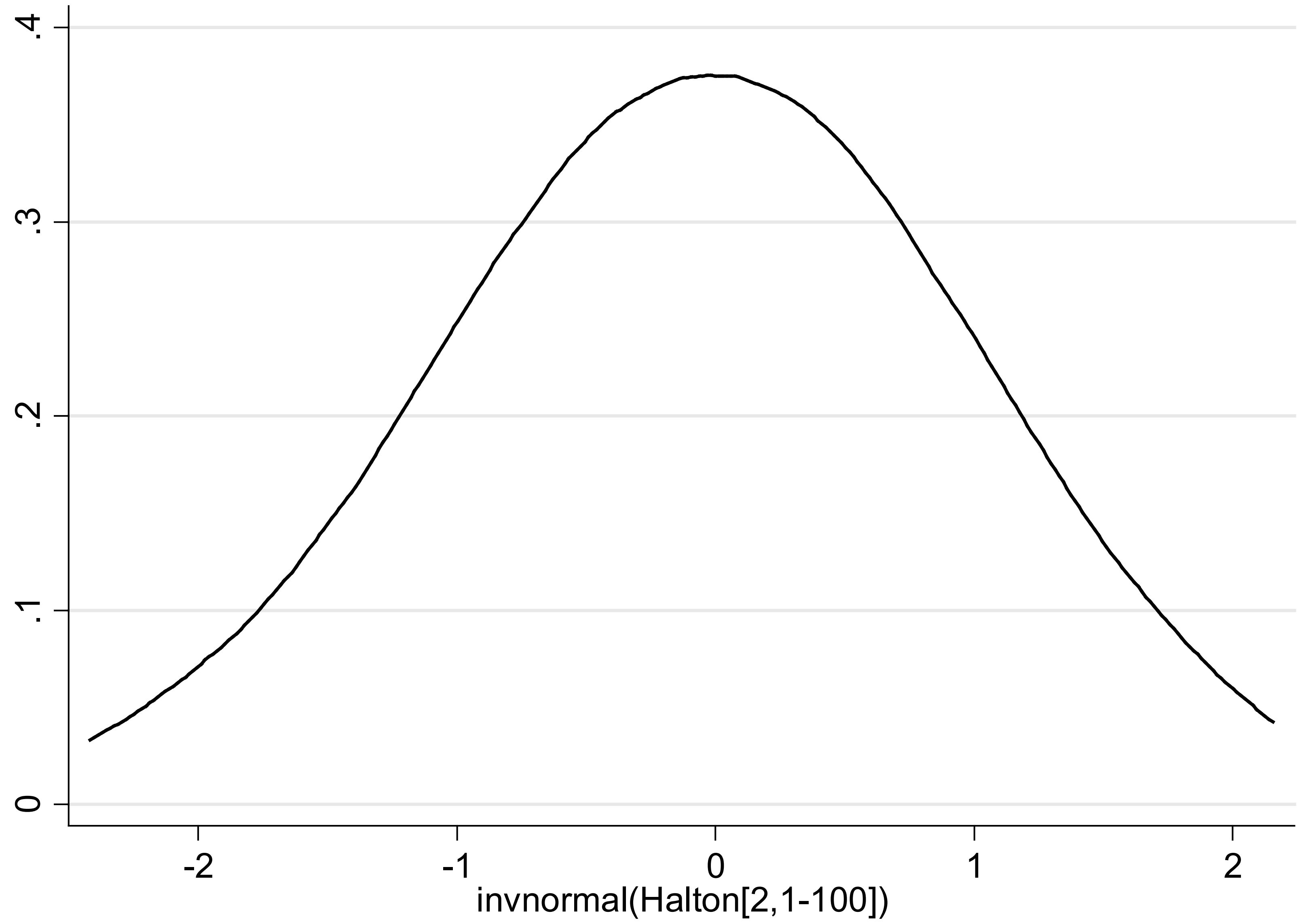
Maximum Simulated Likelihood:

- Concept of MSL:
  - Sample size: $N \times T$
  - k-parameters that need to be integrated out
  - For each parameter, take $d_{ik} = 1, \ldots, D$ draws from a standard uniform density
  - Transform by the inverse standard normal distribution: $\Phi^{-1}(d_{ik})$
  - Calculating the likelihood and averaging over all draws
- Requirement: draws are equally distributed within and between individuals
- In most cases: $N > D$, therefore ensuring equal distribution within i is challenging

Halton sequence:

- Halton sequences have certain characteristics that make them favourable compared to pseudorandom number generator
- Halton sequence are based on prime numbers
- K prime numbers are required (2,3,5,7,…)
- A sequence consists of the first $N \times D$ entries (excluding burned)
- Example for prime number 2:
  - ➢ $\boxed{1/2}, \boxed{1/4}, 2/4, \boxed{3/4}, \boxed{1/8}, 2/8, \boxed{3/8}, 4/8, \boxed{5/8}, 6/8, \boxed{7/8}, \dots$

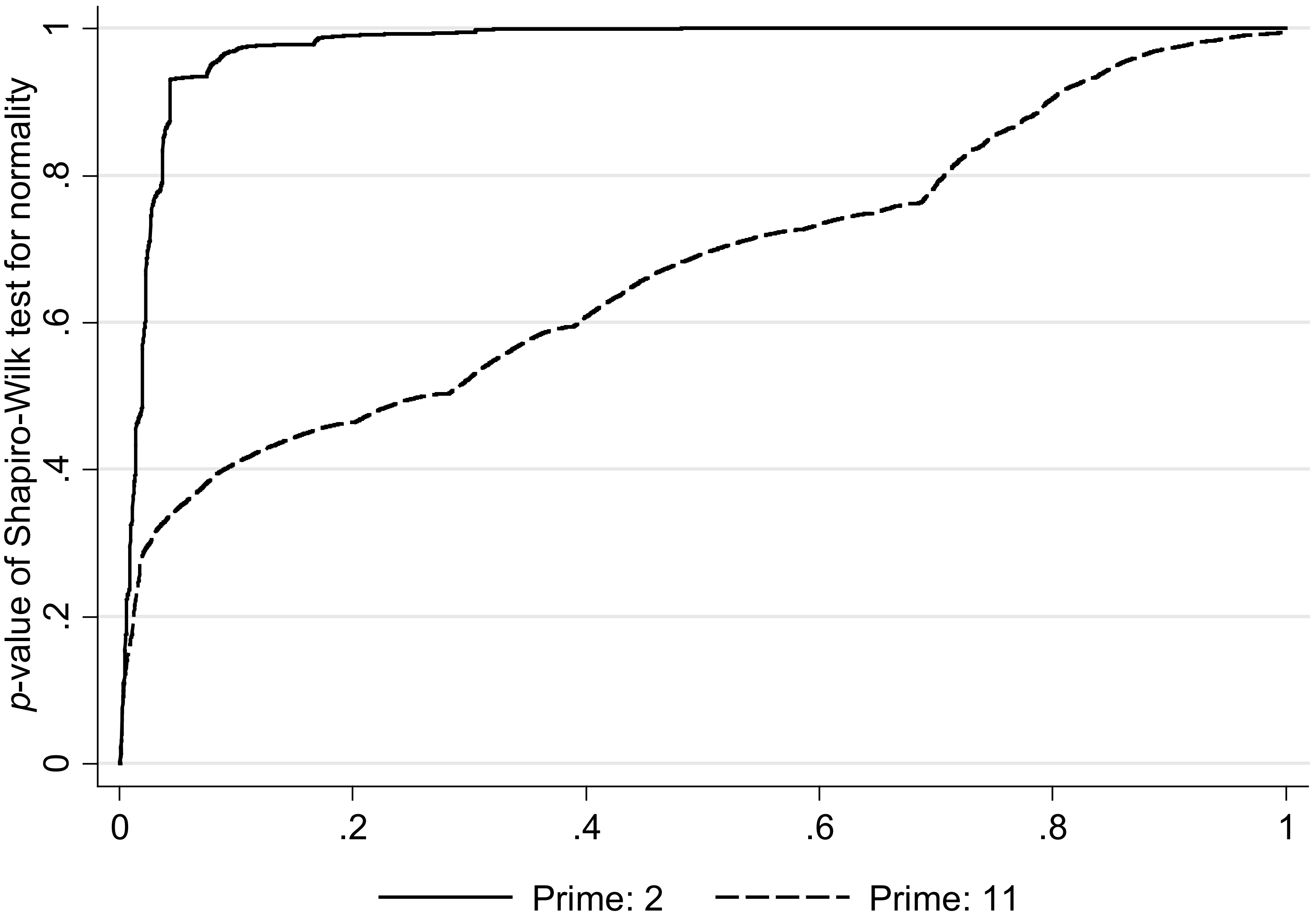
3. Halton draws



Shapiro-Wilk tests for normality:

- Prime numbers 2 & 11
- $N = 5{,}000; D = 50$
- For each individual, applying Shapiro-Wilk test for normality
- Reporting distribution of $p -$ values

3. Halton draws



Randomly assigned normally distributed draws (RAND):

1. Generate D × 2 matrix v_{ki}:

$$v_{ki} = \begin{pmatrix} \underbrace{\Phi^{-1}\big(1/(D+1)\big)}_{a} + \underbrace{(0.5 - r_{11}) * .001}_{b} & \underbrace{r_{21}}_{c} \\ \vdots & \vdots \\ \Phi^{-1}\big(D/(D+1)\big) + (0.5 - r_{1D}) * .001 & r_{2D} \end{pmatrix}$$

where r_{jd} with $d = 1, \ldots, D$ and $j = \{1,2\}$ are random standard normal distributed numbers

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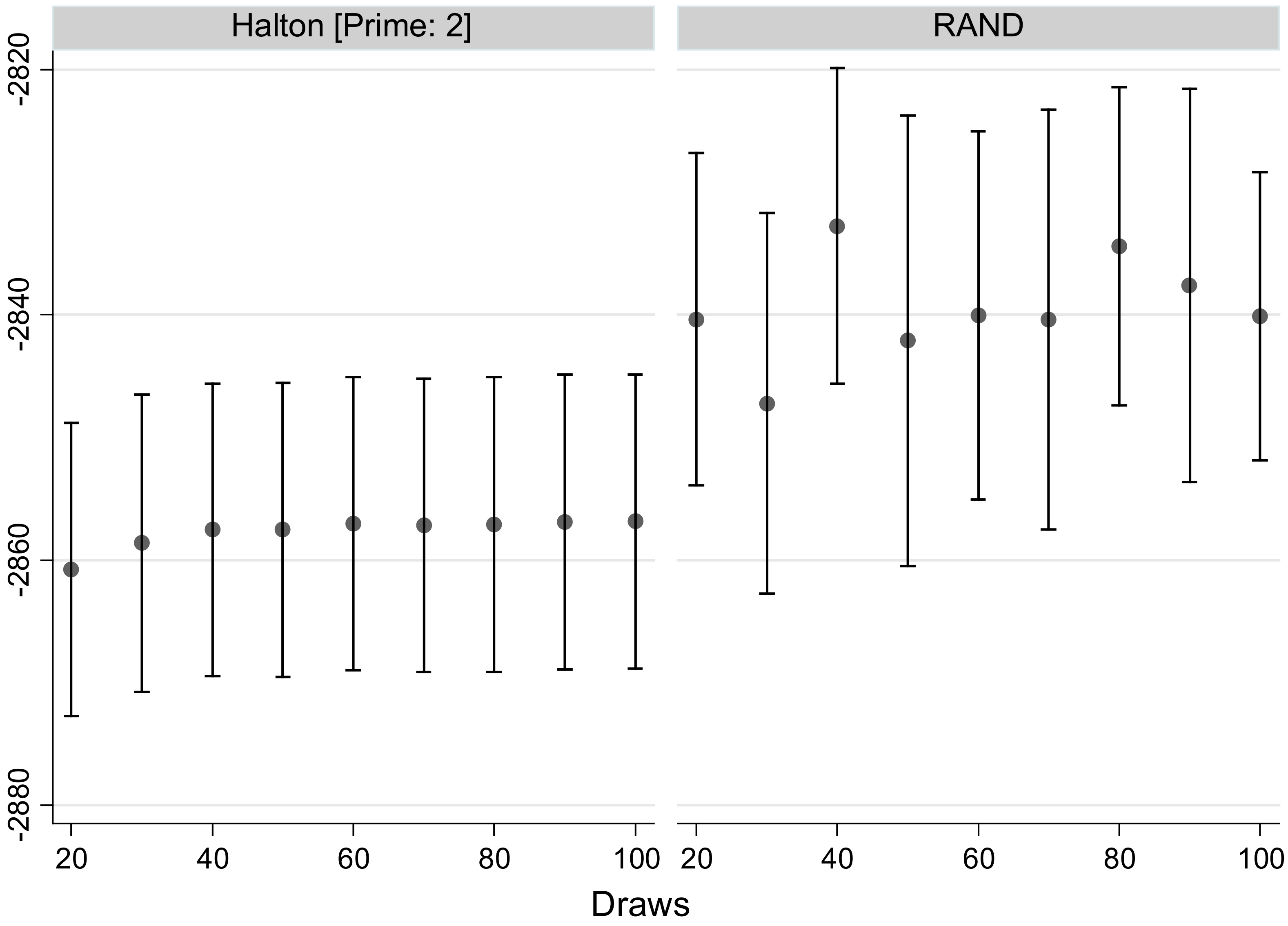
   where r_{jd} with $d = 1, \dots, D$ and $j = \{1,2\}$ are random standard normal distributed numbers
2. Sort v_{ki} according to r_{2d} in ascending order
3. Generate $\text{RAND}_k = (v'_{k1}[1 \dots D, 1] \backslash v'_{k2}[1 \dots D, 1] \backslash \dots \backslash v'_{kN}[1 \dots D, 1])$

Simulation I:

- Univariate equation, $N = 1000$ and $\mathrm{T} = 6$
- 50 replications, $D = \{20,30, \dots, 100\}$
- Halton draws (prime 2) & RAND
- $y_{it} = \mathbf{1}(0.5\tau_{it} + 1 + \alpha_i + u_{it} > 0)$ with $\tau_{it}, u_{it}, \alpha_i \sim N(0,1)$

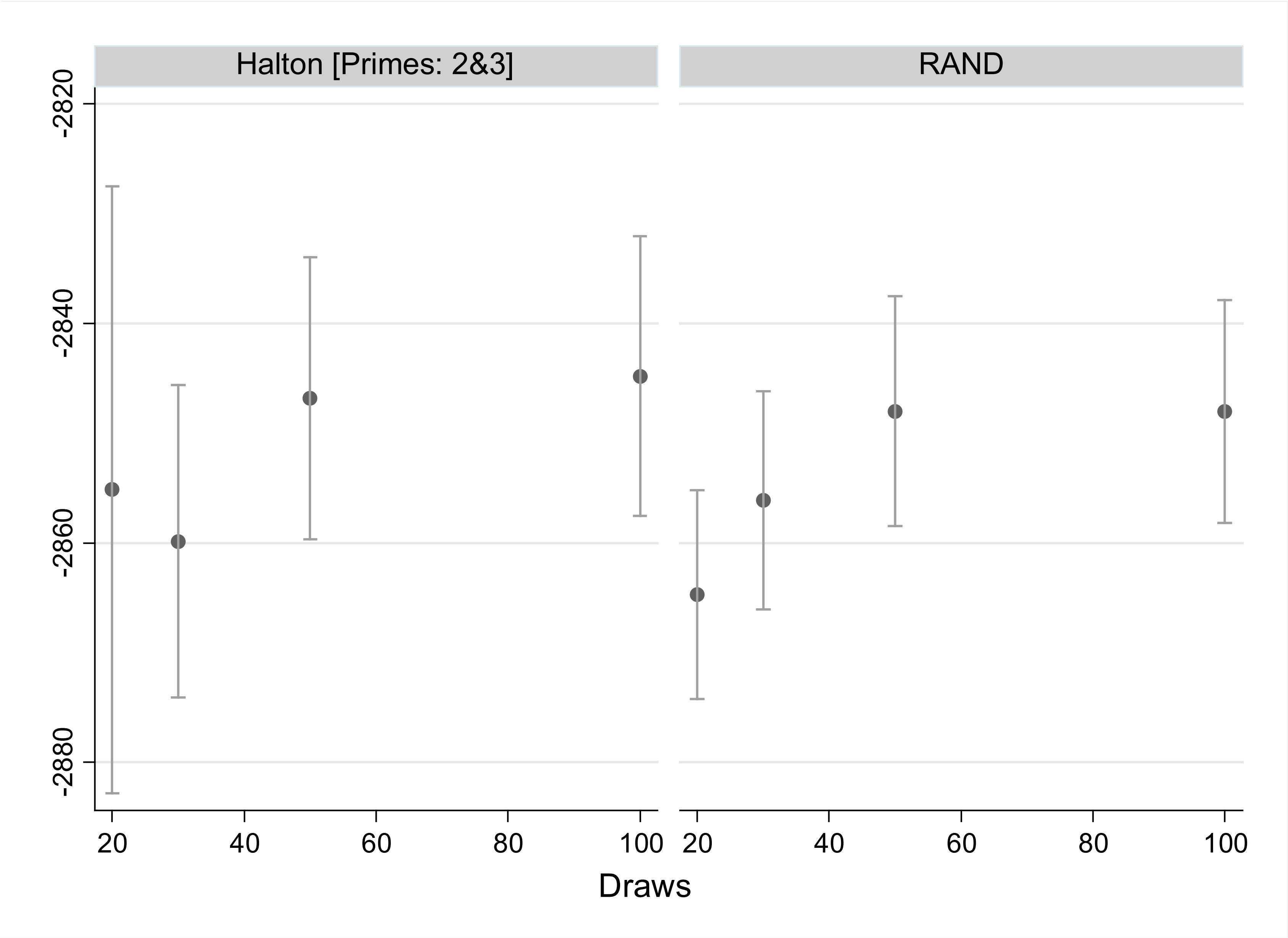
5. Simulation



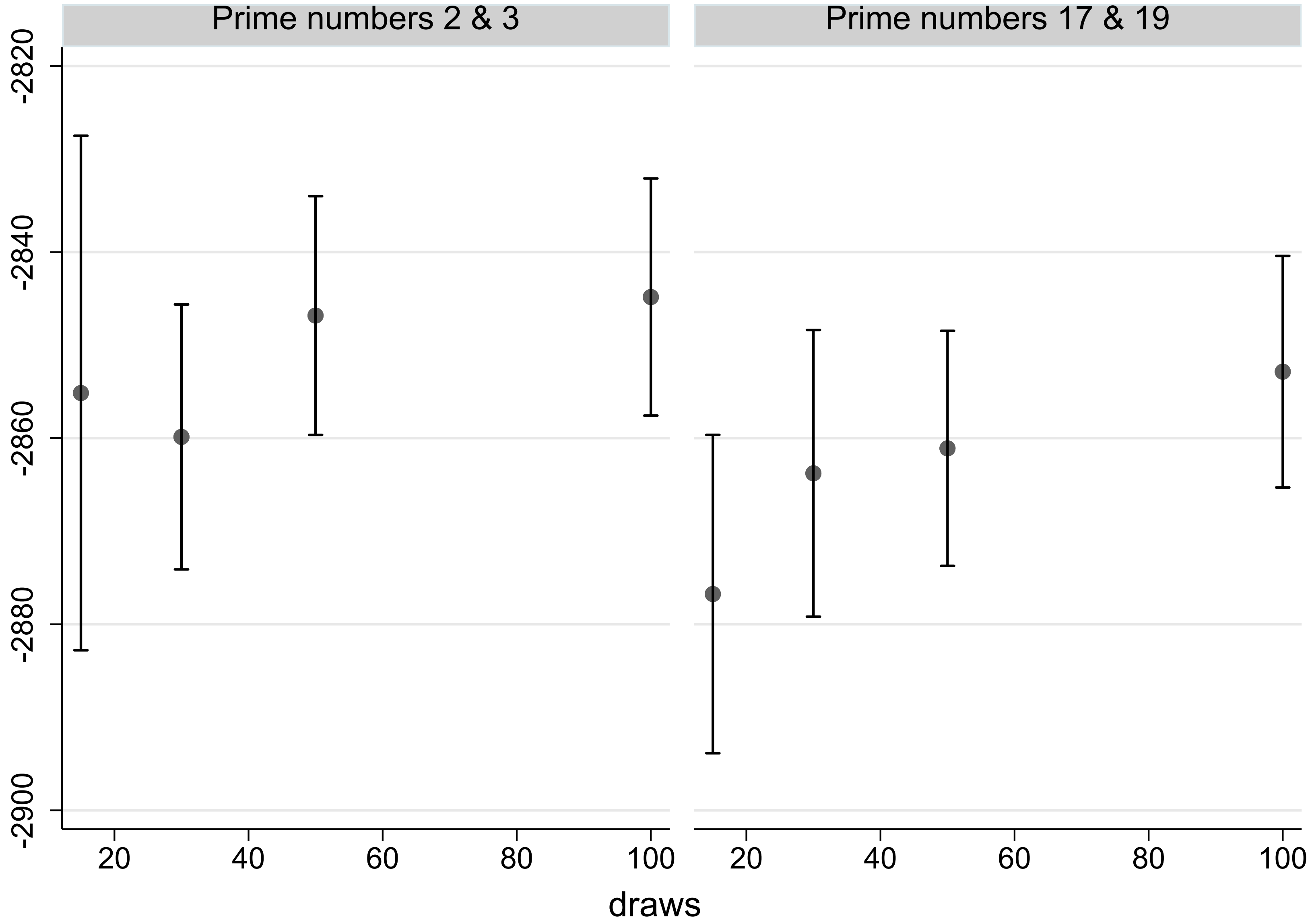
Simulation II:

- Bivariate equation, $N = 500$ and $\mathrm{T} = 6$
- 50 replications, $D = \{15,30,50,100\}$
- Halton draws (prime 2&3) & RAND
- $y_{jit} = \mathbf{1}\left(1 + \alpha_{ji} + u_{jit} > 0\right)$ with $u_{jit} \sim N(0,1)$, $j \epsilon \{1,2\}$ and $VCV = \begin{pmatrix} \sigma^2_{\alpha_1} & \\ \rho\sigma_{\alpha_1}\sigma_{\alpha_2} & \sigma^2_{\alpha_1} \end{pmatrix}$ and $\sigma^2_{\alpha_j} = 1$ and ρ=.8

5. Simulation



5. Simulation



Empirical example:

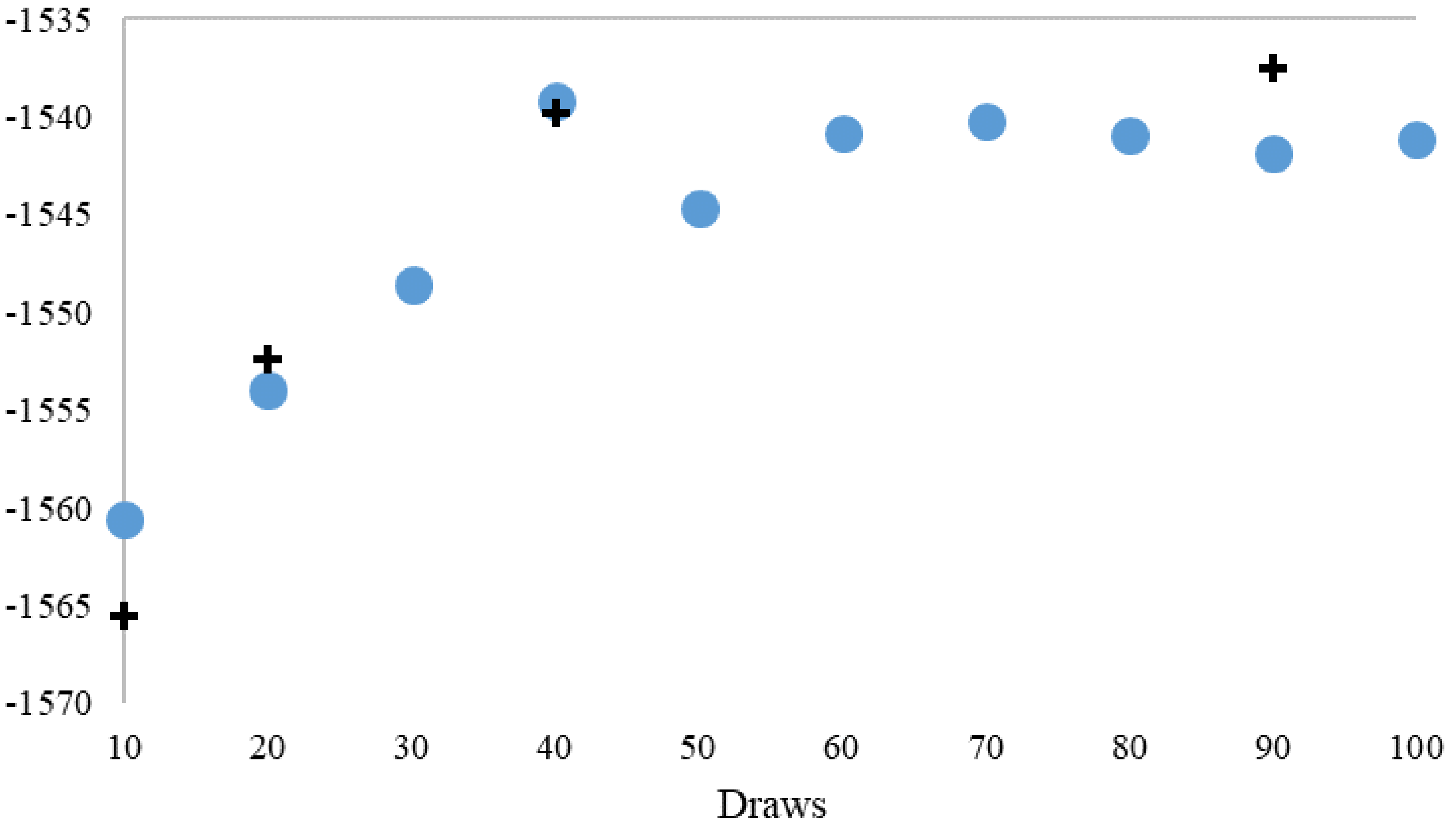
- Cappellari & Jenkins (2007)
- MSL on multivariate normal probabilities
- Probability being unemployed:

$$P_{it} = \Phi^5(x'_{i1}\beta, x'_{i2}\beta, x'_{i3}\beta, x'_{i4}\beta, x'_{i5}\beta)$$

- VCV almost unspecified, 14 parameters estimated $\left(\sigma^2_{\alpha_1} = 1\right)$:

$$VCV = \begin{pmatrix} \sigma^2_{\alpha_1} & & \\ \vdots & \ddots & \\ \rho_{15}\sigma_{\alpha_1}\sigma_{\alpha_5} & \cdots & \sigma^2_{\alpha_2} \end{pmatrix}$$

6. Empirical example (MVNP)



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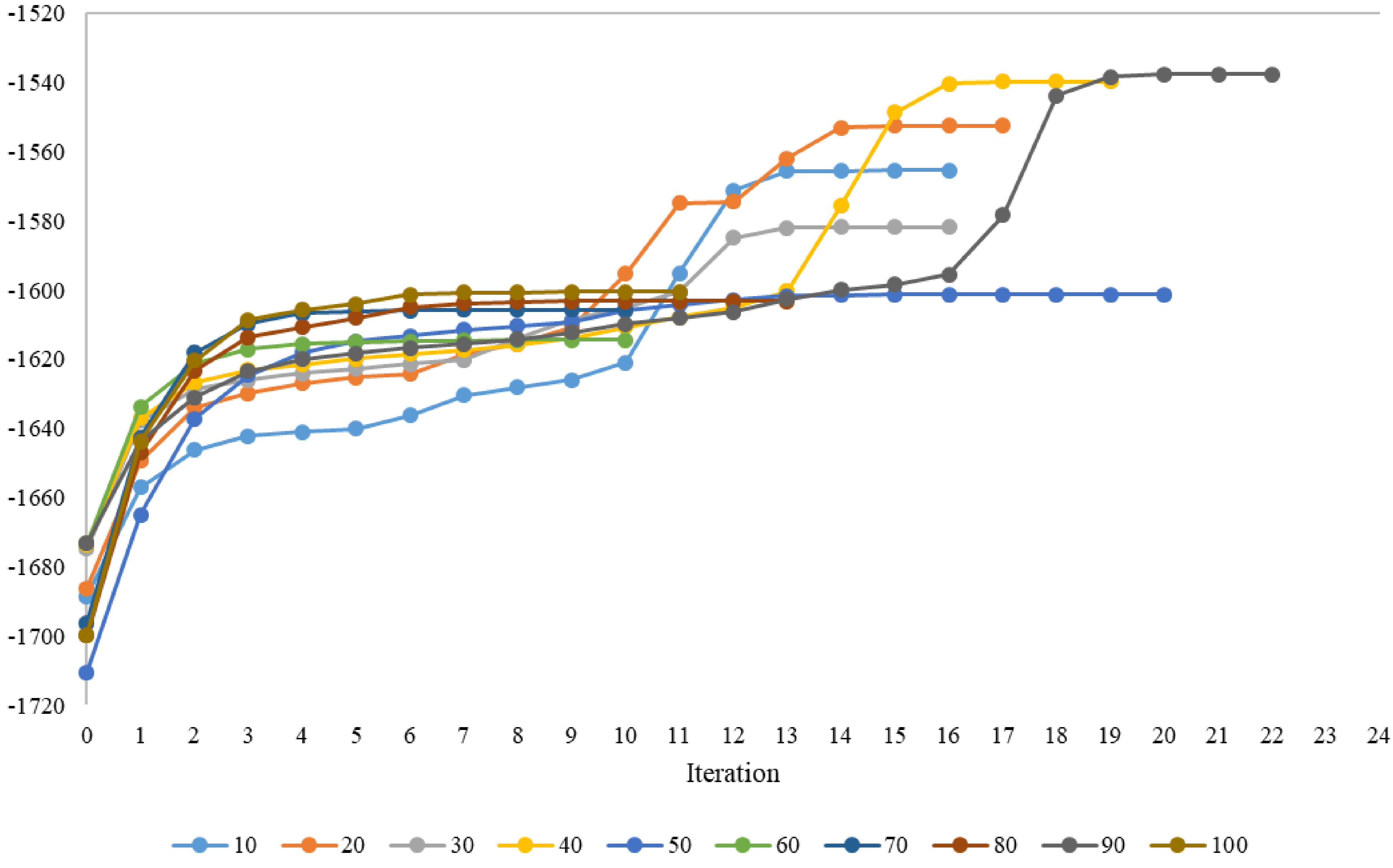
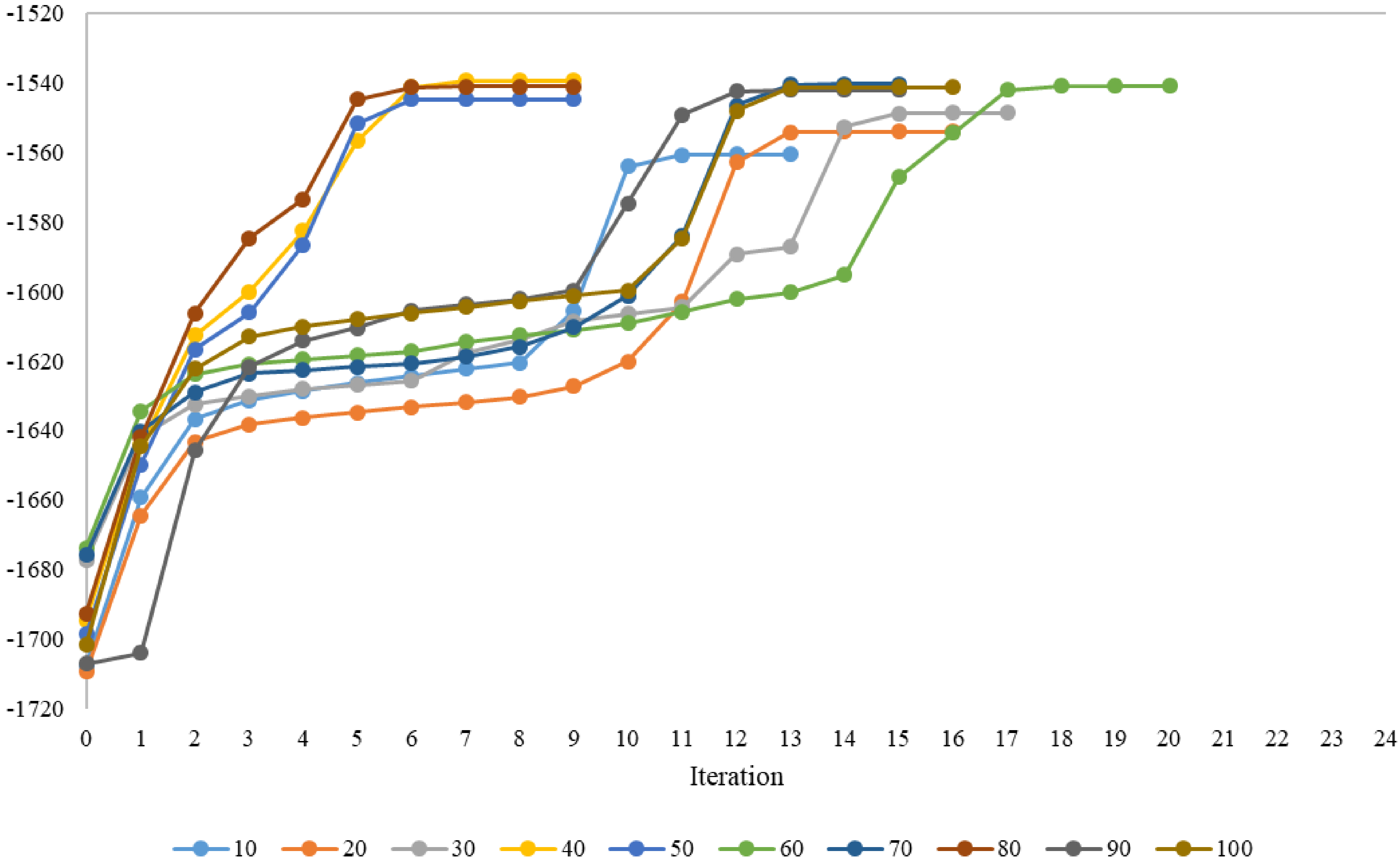


6. Empirical example (MVNP)



Conclusion:

- MSL helpful to integrate out multiple integrals
- Advantage is flexibility in modelling multivariate normal densities
- MSL uses random draws from standard uniform density
- Requirement: Equal distribution within and between individuals
- Quasi-random number generator using prime numbers
- Here: Randomly assigned normally distributed draws

Thank you for your attention!